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Research articles

Efforts to overcome students' learning difficulties in geometry: A didactic design of creative thinking skills through metacognitive approaches

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Abstrak Pengetahuan awal merupakan hal yang paling krusial dalam menghubungkan seluruh informasi yang ada sehingga pengetahuan baru dapat dikonstruksi melalui proses asimilasi atau akomodasi. Tujuan dari penelitian ini adalah (1) untuk mengidentifikasi pengetahuan awal yang dilupakan mahasiswa sehingga mereka kesulitan dalam menyelesaikan soal luas bangun datar, (2) mendeskripsikan lintasan pembelajaran berpikir kreatif berbasis metakognisi, dan (3) untuk mengetahui desain didaktik yang dapat mengurangi kesulitan mahasiswa dalam menyelesaikan masalah penalaran kreatif matematis pada materi luas bangun datar. Metode penelitian ini merupakan metode penelitian desain didaktik dengan dua kali uji coba sebagai cara untuk menjawab rumusan masalah sehingga tujuan penelitian tercapai. Hasil penelitian menunjukkan bahwa mahasiswa masih belum kreatif atau belum bisa keluar dari pengetahuan awal terkait pola bangun datar. Berbekal pengetahuan awal yang ada, mahasiswa belum mampu menganalisis permasalahan bangun datar ke dalam bagian-bagian sederhana serta mensintesis kembali bagian-bagian tersebut ke dalam bentuk yang lebih kompleks (analisis dan sintesis). Selain itu, mahasiswa belum mampu mengurai bangun datar menjadi segitiga atau segi empat yang lebih kecil dan menghitung luas atau kelilingnya serta belum memahami hubungan antara berbagai bangun datar (relasi antara bangun datar). Untuk mengantisipasi permasalahan tersebut, lintasan pembelajaran berpikir kreatif matematis berbasis metakognisi telah dikembangkan yang terdiri atas lima tahapan yaitu orientasi pada masalah, rencana mengatasi masalah, realisasi rencana, penguasaan pengetahuan matematika awal (konsep kreativitas), dan evaluasi hasil yang diperoleh. Berdasarkan hasil ujicoba, rangkaian desain didaktik yang dikembangkan dapat mengurangi kesulitan yang dihadapi mahasiswa.

Kata kunci *Desain Didaktik, Metakognisi, Pengetahuan awal, Keterampilan berpikir kreatif, segi empat.*

Abstract Prior knowledge is crucial in connecting all available information so that new knowledge can be constructed through the processes of assimilation or accommodation. The objectives of this study are: (1) to identify the prior knowledge forgotten by students that causes difficulties in solving problems related to the area of Euclidean plane; (2) to describe the learning trajectory of creative thinking based on metacognition; and (3) to determine a didactic design that can reduce students' difficulties in solving problems involving mathematical creative reasoning on the topic of Euclidean plane. This research uses a didactic design research method with two trials as a means to address the research questions and achieve the research objectives. The results show that the college students are still not creative or unable to move beyond their prior knowledge related to the patterns of the Euclidean plane. With their existing prior knowledge, the students are not able to analyze problems involving Euclidean plane into simpler parts, nor can they synthesize these parts into more complex forms (analysis and synthesis). More specifically, the students are unable to

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decompose Euclidean planes into smaller shapes of triangles and quadrilaterals as well as calculate their area or perimeter, nor do they understand the relationships between different shapes of Euclidean planes. To address these issues, a learning trajectory of creative mathematical thinking through metacognitive approaches has been developed, consisting of five stages: problem orientation, planning to solve the problem, realization of the plan, mastery of prior mathematical knowledge (concept of creativity), and evaluation of the solutions. Based on the trial outcomes, the developed didactic design sequence can reduce the difficulties faced by students.

Keywords *Didactic Design, Metacognition, Prior knowledge, Creative, Quadrangular thinking skills*

Introduction

The limited mathematical creative reasoning abilities of students in comprehending geometry pose a significant challenge in higher education. Geometry necessitates the visualization of shapes and structures, which is also a crucial component of creative reasoning. The capacity to envision alterations in form and space is fundamental to numerous creative processes. Geometry frequently entails intricate problems that necessitate unconventional thinking to resolve. It fosters the utilization of creative reasoning to discover indirect or intuitive solutions.

Geometry and creative reasoning abilities are two fundamental components of education and cognitive development that are intricately interconnected and mutually supportive. Geometry provides the structural and logical framework necessary for comprehending the physical world, while creative reasoning enables the utilization and manipulation of geometric concepts to generate innovative and aesthetically pleasing solutions. The synergistic combination of these two abilities is indispensable for solving intricate problems, adapting to change, and fostering innovations across diverse domains. The close relationship between these abilities creates a gap that arises from a lack of understanding regarding the effective application of creative reasoning in geometry education, particularly within the broad scope of quadrilaterals. While numerous studies have investigated metacognition learning, there is a dearth of research specifically examining how metacognition learning can be tailored to address geometry learning challenges.

Geometry is a fundamental branch of mathematics that holds significant relevance in our daily lives. Its connections extend beyond mathematics, intertwining with other disciplines and fields. As elucidated by Biber et al. (2013), geometry serves as a bridge between everyday occurrences and mathematical concepts, thereby emphasizing its pivotal role in the study of mathematics. This notion aligns with the perspective of Zuya and Kwalat (2015), who assert that "problems arising from other branches of mathematics can be resolved through the application of geometric knowledge, extending beyond its practical applications in everyday life." Consider, for instance, a quadratic function $f(x) = x^2$. We seek to determine the area beneath this curve from the specified point $x = 0$ to $x = 2$. This approach employs a geometric methodology. In fact, this geometric methodology forms the foundation of numerous concepts in calculus, where integrals are employed to determine the area beneath a curve. Although the integral is frequently regarded as a calculus concept, its origins lie in a geometric context, specifically in calculating the area under a curve.

Consequently, a fundamental comprehension of geometry facilitates the comprehension and resolution of problems pertaining to calculus. This exemplifies the interconnectedness and solvability of various mathematical disciplines, such as calculus, through the utilization of geometric knowledge. Geometry serves as a foundational methodology employed by individuals to comprehend and elucidate the physical environment, involving measurements of length, surface area, and volume. Notably, geometry encompasses a captivating aspect of mathematics, presenting numerous intriguing approaches and challenges. Furthermore, geometry engages the visual, aesthetic, and intuitive senses (Ganal & Guiab, 2014).

As elucidated in the provided explanation, individuals pursuing further studies in geometry must possess creative and original reasoning abilities. When confronted with challenges, individuals with robust comprehension skills effortlessly grasp concepts and adeptly establish innovative connections between these concepts to identify solutions. Geometry students are expected to defend their solutions to problems (Verner, Massarwe, & Bshouty, 2019). However, learning difficulties in geometry are primarily attributed to the state of mental models employed by students, the strategies they utilize, and the active schemes during the problem-solving process. Similarly, students often struggle to comprehend fundamental concepts in geometry and may engage in studying without a thorough understanding of basic terminology (Halat, 2008). This phenomenon was also observed by Burger and Shaughnessy (1986), who noted that students encountered difficulties in identifying images and proving theorems in geometric constructs.

In the context of the development of Pedagogical Didactic Anticipation (ADP), there are (learning obstacles), particularly those that are (epistemological obstacles). In the field of geometry, epistemological obstacles frequently arise due to disparities between students' intuitive comprehension and the formal concepts introduced in the learning process. Here are some examples of epistemological obstacles in geometry. Students are perplexed by angles exceeding 180 degrees or the concept of negative angles. This difficulty is associated with the transition from visual and intuitive understanding to a more formal and quantitative comprehension.

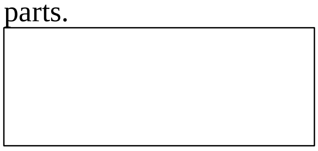
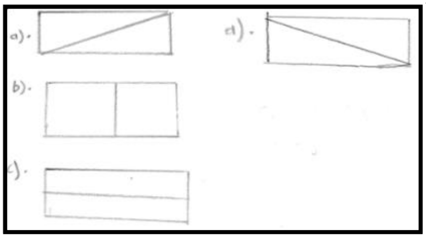
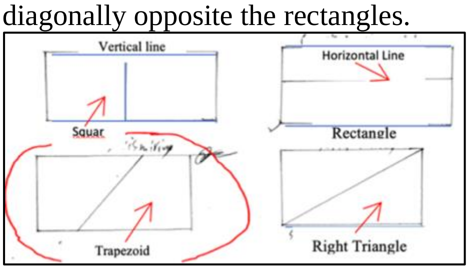
Learning obstacles are symptoms that manifest in students, characterized by lower learning outcomes compared to their previous achievements. Consequently, learning obstacles are a condition within the learning process that entails specific impediments to achieving learning outcomes. For students, if you possess a desire or aspiration, it is imperative to pursue it with unwavering determination, accuracy, and patience (Didik, 2017; Kusrianto, 2013; Prahmana & D'Ambrosio, 2020).

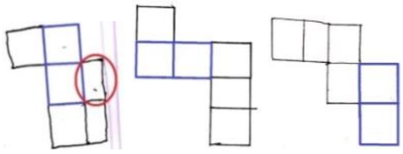
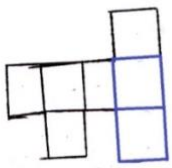
According to Wallas (2014), the creative thinking process comprises four distinct stages: the preparation stage, the incubation stage, the illumination stage, and the verification stage. This stage is referred to as the Hypothetical Learning Trajectory (HLT), which represents a hypothetical learning path. It entails hypothesizing a series of activities that students typically engage in while solving problems or comprehending concepts. The flow derived from multiple revisions is designated as the learning flow (learning trajectory).

In contrast, the student learning path emphasizes the hypothetical plan devised by educators to guide students' learning journey from their initial understanding to a more sophisticated and intricate comprehension of a concept or skill. On the other hand, the creative thinking process focuses on generating novel, original, and beneficial ideas. It also entails considering various perspectives and identifying innovative solutions to challenges.

A preliminary study was conducted by researchers in lecture halls to investigate mental models, the strategies employed by students, and the active schemes during the problem-solving process. Table 1 presents an example of diagnostic questions and their corresponding solutions conducted by students.

Table 1. Examples of student creative thinking questions and solving

No	Student inquiries and responses pertaining to the realm of creative thinking
1	Divide a rectangular shape into two equal parts using a ruler, scissors, or any other appropriate tool. Make the division as precise as possible and draw the resulting two equal parts.  Pattern and variation in student responses  In the responses to parts b) and c), students continue to divide flat images into two rectangular parts of equal area or shape, achieved through two methods: horizontally and vertically, or folding. This approach demonstrates a lack of creativity and adherence to the existing flat building pattern. Students are unable to break free from this pattern and explore alternative solutions. Students are closely associated with the shape and flat building pattern that mirrors the existing problem. Their initial knowledge often fails to analyse flat shapes into their component parts and synthesize these parts into complex forms (analysis and synthesis). Initial knowledge reduces flat shapes into smaller triangles or rectangles and calculates their area or circumference. However, students often lack an understanding of the relationships between various flat shapes. In contrast, the solutions to parts a) and d) involve dividing the rectangle into two parts of a right triangle of equal area or shape. This approach not only demonstrates fluency but also introduces flexibility by allowing for two ways of folding or cutting both ends (corners) diagonally opposite the rectangles.  Another finding is the division of the rectangle into two trapezoidal parts of equal area, shape, and size, achieved through two methods: folding or dividing the paper obliquely. Even without novelty value, students demonstrate creativity. They can deviate from or adhere to the available flat drawing images. The outcomes of the students' responses are influenced by the presence of the student worksheet in the design of the didactic situation analysis prior to learning. This worksheet, in the form of Hypothetical Didactic Design including Pedagogic Didactic Anticipation (ADP), facilitates the development of a didactic design that can reduce the learning gap faced by students. The students in this study exhibit problem-solving abilities. They successfully plan to solve problems by drawing according to the problem's

No	Student inquiries and responses pertaining to the realm of creative thinking
	requirements, executing the plan in accordance with the problem, and mastering prior knowledge (the concept of creativity) in mathematics. Additionally, they evaluate the results obtained by dividing the rectangle into two conformal trapezoidal flat shapes.
2	Construct a grid of cubes, utilizing the following two squares as reference points.  Pattern and variation in student responses  (a) (b) (c) Errors identified: (a) Students lack the understanding of placing a circled square on the picture, resulting in the nets not forming cubes. (b) and (c) demonstrate students' lack of meticulous verification of the accuracy of their nets. If folded according to the ribs, the cube formed will have one side doubled and the other side empty.  (d) Part (d) of the findings reveals that students draw seven squares on the nets, despite the cube consisting of only six square-shaped sides. This results in the cube being formed having one double side. Based on the findings of students' creative thinking skills on cube net material, it is evident that their mathematical reasoning is still hindered when verifying their answers. This is particularly evident in the responses of students attempting to find cube webs with original shapes. While they have progressed beyond the illumination stage, they lack the necessary rigor to check their answers against reality. From the overall test results, the maximum number of distinct cube nets that students can identify per individual is eight. However, when considering all student answers, all cube nets can be identified, totalling eleven pieces. Notably, these eleven cube webs were discovered by different students.

Through an analysis of the responses, it was identified that students lacked creativity and were unable to break free from the conventional flat building pattern. Students are heavily influenced by the shapes and drawings of flat buildings that closely resemble the available questions, lacking any novelty. This has hindered their creative reasoning when verifying their answers. Students' learning trajectory is already oriented towards the problem, with a plan to overcome it, the realization of that plan, the mastery of prior knowledge, and the evaluation of the results obtained. Metacognition plays a crucial role in students' understanding and problem-solving abilities, particularly in the context of mathematics. This includes the capacity for students to plan, monitor, and evaluate their approach to mathematical problems. For instance, when calculating the area of a rectangle, students can plan to divide the shape into simpler parts (e.g., dividing a rectangle into two triangles). Similarly, when controlling the lengths of the sides of a quadrilateral, students must ensure that all measurements are accurate and consistent with the properties of the quadrilateral. Additionally, they should verify whether their method is correct and whether the final result aligns with their expectations or can be recalculated using a different method for verification.

The five trajectory points identified in this study that outline the student creative learning trajectory are: (1) Orientation to Problems: This point encompasses the student's initial identification and understanding of the creative challenges they face; (2) Planning to Overcome Problems: In this phase, students develop a comprehensive plan of action to address the identified problems; (3) Realization of Plans: This point involves the implementation of the student's plan and the successful execution of their creative strategies. (4) Mastery of Previous Knowledge (Concepts of Creativity): Students demonstrate a deep understanding and application of creative concepts, including mathematical principles, during this stage; and (5) Evaluation of Results Obtained: This point involves assessing the effectiveness and outcomes of the student's creative endeavors.

Throughout the creative learning trajectory, students engage in metacognition, which involves reflecting on their learning process. This entails evaluating their planning, executing their actions, and formulating and selecting creative ideas.

This study's findings suggest that problem orientation should be the initial step in students' creative mathematics learning trajectory. Individuals who engage in problem orientation during the creative thinking process are the first to recognize problems. Additionally, the study identified students' metacognition activities that align with Schoenfeld's (1992) theory, which proposes three methods of metacognition in mathematics learning: belief or intuition, knowledge of the thought process, and self-awareness in independence. Individuals' beliefs significantly influence their problem-solving strategies, while knowledge of thought processes indicates their ability to effectively utilize their thought processes.

According to Supriatna (2011) and Irawan (2015), the development of didactic design can contribute to the development of knowledge in the field of mathematics and serve as a tool to facilitate learning. Furthermore, metacognition skills are considered the most challenging thinking skills (high other thinking skill category) due to their complexity and intricacy. Metacognition involves not only the ability and understanding of knowledge aspects but also other abilities such as cognitive abilities, writing proficiency, and an understanding of the reality to be expressed and the motivation for reasoning. On the other hand, thinking skills play a crucial role for students in both academic and professional pursuits, whether during their academic years or when they enter the workforce. Third, in the context of learning with a metacognition approach, student activity sheets that provide scaffolding not only develop metacognition skills in expressing creative ideas in writing but also actively engage students in the learning process of planning, controlling, and evaluating. This approach bridges their academic needs and effectively addresses learning obstacles, student learning trajectories, and student characteristics. Consequently, the development of didactic design holds significant influence on how students learn in the classroom (Suryadi, 2010). Even the development of new theories is anticipated to provide solutions to learning obstacles, student learning trajectories, and student characteristics.

Consequently, researchers are interested in exploring the role of DDR in developing and testing effective learning interventions within the classroom setting. For instance, in the realm of geometry, where conceptual understanding frequently necessitates visual representation and manipulation of tangible objects, the metacognitive approach facilitates students' enhanced awareness and independence. By integrating these two approaches, a more comprehensive geometry curriculum can be developed, thereby supporting the attainment of superior learning outcomes for students.

The continuous development of didactic design requires collaborative efforts from both educators and researchers. As outlined by Kansanen (2003), mathematics learning encompasses two fundamental aspects: the relationship between students and the material, and the relationship between students and teachers. The latter is referred to as the pedagogical relation, while the former is known as the didactical relation (HD). This didactical relation is typically depicted in didactic triangles. The primary objectives of this study are to: (1) identify the initial knowledge gaps among students that hinder their ability to solve problems involving flat building areas, (2) ascertain the learning trajectory of students based on a rectangular context through the application of Metacognition, and (3) explore the didactic design elements that influence the reduced difficulties experienced by students in solving mathematical creative reasoning problems involving flat building materials.

Methods

This research incorporates Didactical Design Research conducted through a qualitative methodology (Suryadi, 2013). The objective is to address student learning obstacles in comprehending the concept of Euclidean plane. This study elucidates the challenges encountered by students and proposes solutions to overcome them. The obstacles identified by students serve as the foundation for designing teaching materials tailored to their needs. The research implementation process is structured into three distinct stages.

The initial stage entails analyzing the didactic design prior to learning, encompassing the selection and determination of mathematics materials that will serve as research materials from the student's handbook, study materials that will also serve as research materials, discussions with mathematics partner lecturers, the selection and determination of test questions, specifically student ability test questions that encompass a diverse range of questions that pose challenges to student learning, the implementation of test 1 for all students in grades D, E, and class F, and conducted interviews using purposive sampling techniques. A sample of six students (two students in the good category, two students in the middle category, and two students in the low category) were interviewed for each question item. The test results 1 and interview results were subsequently analyzed, and didactic designs were compiled in accordance with student learning obstacles in solving questions. The second stage involves metapedadidactics analysis, which entails applying the didactic design to 28 students. The third stage entails retrospective analysis, encompassing the analysis of the extent to which the didactic design was implemented with learning metacognition approaches that occur when the didactic design is applied, conducting test 2 with students and conducting interviews, and subsequently analyzing the results. From these three stages, empirical didactic design will be obtained, which is capable of continuous refinement through the iterative process of DDR.

This study's primary objective is to elucidate the research's purpose. It commences with an analysis of the didactic environment, specifically the didactic relationship between the student and the material, which is manifested in the capacity for creative thinking. The anticipated outcome of this study is the development of didactic design, a teaching resource that underscores creative thinking across a broad spectrum of Euclidean plane concepts. An overview of the didactic design is presented in Figure 1.

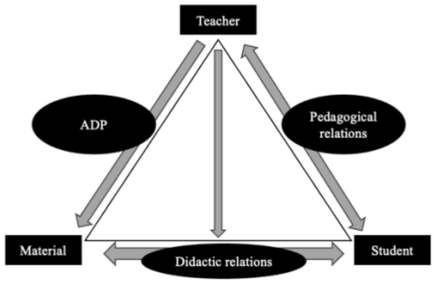


Figure 1. Didactic design research scheme (Suryadi, 2019)

The didactic relationship between students and teaching materials necessitates students' metacognitive abilities in comprehending and solving mathematical problems. By incorporating metacognition into didactic design, students are equipped with the knowledge and skills to plan, control, and evaluate their learning and understanding of the broad concept of the quadrilateral. This includes developing fluency, flexibility, and originality in their creative thinking. Furthermore, students are empowered to make the necessary adjustments to enhance their creative thinking abilities.

The instruments employed in this study comprise two creative thinking questions, a questionnaire, and an observation sheet interview. Two creative thinking questions were meticulously designed to assess students' proficiency in solving geometry problems. The instrument's construction was guided by indicators of creative thinking skills, specifically fluency, flexibility, and originality. This instrument serves as a measure of the extent to which metacognition-based learning and didactic design facilitate students' development of these skills. The instrument's content validity was rigorously evaluated by three mathematics education experts through in-depth discussions. They scrutinized the suitability of the questions with respect to the indicators of creative thinking and the objectives of geometry learning. The instrument underwent testing on a small sample (a class that did not constitute the primary subject of the study) to identify potential technical difficulties or concepts that students may have encountered. Based on student feedback and the outcomes of statistical analysis, improvements were implemented. Construct validity was assessed through exploratory factor analysis (EFA). The results revealed that all questions possessed a factor loading value exceeding 0.60, indicating statistical validity for measuring creative thinking skills. Reliability was determined using the Cronbach's Alpha coefficient, which yielded a result of 0.85 (high category), suggesting substantial internal consistency. Consequently, this instrument has met the validity and reliability criteria, rendering it suitable for assessing students' creative thinking abilities. The outcomes of this assessment will serve as a foundational basis for evaluating the efficacy of the metacognitive approach and didactic design in enhancing students' creative thinking skills within the context of geometry learning. The two questions in question are presented in the following Table 2.

The questionnaire was employed to assess students' perceptions of the learning experience employing the metacognitive approach and the didactic design implemented (as outlined in Table 3). The questionnaire's questions were structured using a five-point Likert scale, encompassing aspects such as comprehension of the material, creative thinking abilities, challenges encountered during the learning process, and the effectiveness of the metacognitive approach. The questionnaire's preparation underwent a series of stages: (1) Content Validation: The questionnaire underwent testing by a mathematics education expert to ascertain its suitability with the research objectives. (2) Validity and Reliability Assessment: A small group of students (outside the main sample) was employed to validate the questionnaire's reliability and consistency. (3) Construct Validity Evaluation: Factor analysis techniques were employed to assess construct validity. The Kaiser-Meyer-Olkin (KMO) statistic yielded a value of 0.82 (indicating a "good" category), while the Bartlett test yielded a significant value (< 0.05). (4) Reliability Measurement: Cronbach's Alpha coefficient of 0.87 (classified as "high") was calculated, indicating the questionnaire's consistency in measuring student perceptions.

Table 2. Types of questions presented to respondents

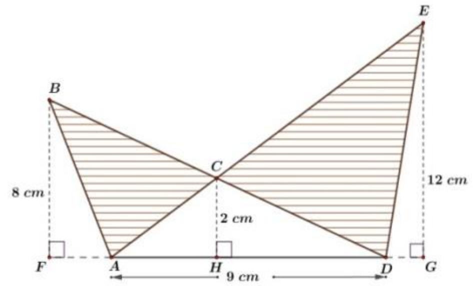
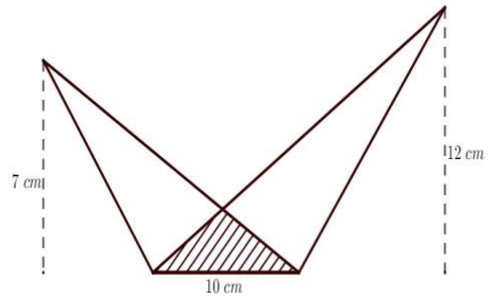
Problem Type	Problems
1	 Please calculate the area of the shaded region. Provide a step-by-step solution to this problem, outlining the sequence of operations and any challenges encountered during the solution process.
2	 If the area of the region not shaded is 55 square centimeters, determine the area of the shaded region.

Table 3. Student perception questionnaire on the learning experience

Aspect	Statement	STS	TS	N	S	SS
Understanding the material	Since completing this course, I have gained a comprehensive understanding of the fundamental principles of geometry.					
	The methodology employed facilitated my comprehension of geometric principles in practical contexts.					
	The comprehension of the taught material is facilitated by the didactic design implemented.					
Creative thinking skills	This educational experience prompted me to explore diverse methodologies for solving geometry-related problems.					
	I possess the ability to generate novel concepts while solving geometry-related problems.					
	The learning process enhances my ability to approach problem-solving with adaptability and creativity.					
Challenges encountered in the learning	Certain sections of the instructional material presented posed challenges in terms of comprehension.					

Aspect	Statement	STS	TS	N	S	SS
process	This educational approach effectively mitigated the challenges I previously encountered in grasping the concepts of geometry.					
	The provided questions were indeed challenging, but I was able to successfully solve them with the appropriate guidance.					
Efficacy of the metacognition approach	The metacognitive approach heightened my awareness of my own thought processes during the learning process.					
	I possess the ability to meticulously plan my approach to problem-solving.					
	I conduct more frequent evaluations of my responses prior to collecting the results.					
	This educational experience has significantly bolstered my confidence in my ability to acquire mathematical knowledge.					

Observation sheets, as outlined in Table 4 and Table 5 serve as comprehensive documentation tools for capturing activities and interactions during the learning process. These sheets facilitate the recording of both the lecturer’s approach (material delivery and instructional strategies) and students’ participation, responses to questions, and creative thinking processes. The observation indicators are meticulously developed based on the creative thinking skills framework, which encompasses fluency, flexibility, and novelty. Additionally, they incorporate metacognitive components such as planning, monitoring, and evaluation.

Table 4. Lecturer activities and interactions

No.	Observation indicators	Checklist		Information
		Yes	No	
1	The lecturer provided the material with clarity and organization.			
2	The lecturer employs a metacognitive approach in delivering the material, involving students in planning their strategies.			
3	Lecturers provide guidance in monitoring students' thinking processes.			
4	The instructor encourages students to critically assess their work.			
5	During the learning process, lecturers offer constructive feedback to students.			

Table 5. Students’ activities and interactions

No.	Observation indicators	Checklist		Information
		Yes	No	
1	Students exhibit proficiency in generating innovative solutions to complex problems.			
2	Students possess the ability to demonstrate adaptability in exploring various problem-solving approaches.			
3	Students are expected to generate distinctive or			

No.	Observation indicators	Checklist		Information
		Yes	No	
	innovative solutions to the problems presented.			
4	Students formulate a plan before commencing problem-solving (planning).			
5	Students diligently monitor their work processes, ensuring that the strategies employed are effective and appropriate.			
6	Upon completing the task, students evaluate the quality of their work.			

Subsequently, interviews were conducted to obtain in-depth information about students' learning experiences during instruction employing a metacognitive approach and didactic design. The interviews were designed to explore students' comprehension of geometry concepts, the challenges they encountered, and their perspectives on the efficacy of learning in enhancing creative thinking abilities. The interview data was integrated with test results, questionnaires, and observations, facilitating data triangulation to refine the didactic design and draw conclusions regarding the impact of the metacognitive approach on students' creative thinking skills in geometry content. The interviews were conducted in two stages: Stage 1, following the implementation of Test 1, to identify students' learning obstacles on geometry problems and their initial understanding of the approach employed. Stage 2, following the implementation of Test 2: To evaluate students' experiences during instruction with the implemented didactic design and ascertain the development of their creative thinking skills.

Data analysis in this study was conducted to integrate findings from tests, questionnaires, observation sheets, and interviews to provide a comprehensive understanding of the effectiveness of metacognitive approaches and didactic design in enhancing students' creative thinking abilities. Data from the tests were analyzed descriptively and quantitatively by comparing the results before (Test 1) and after (Test 2) the implementation of the didactic design. The scores of each creative thinking indicator, namely fluency, flexibility, and novelty, were calculated and averaged, which were subsequently compared to identify any changes that occurred. The results of this test were supplemented with questionnaire data that were analyzed statistically descriptively to describe students' perceptions of learning, encompassing aspects of understanding the material, creative thinking skills, learning difficulties, and the effectiveness of the approach. The findings from the questionnaire were utilized to ascertain the extent to which students' perceptions aligned with the outcomes of their creative thinking skills assessments.

Subsequently, the data from the observation sheet was analyzed to assess the implementation process of the didactic design and metacognitive approach, both from the lecturer's and student's perspectives. Quantitative checklist data was processed to determine the percentage of indicator achievement, while descriptive notes were analyzed thematically to identify patterns of student and lecturer activities, particularly those related to planning, monitoring, and evaluation within the metacognition process. To enhance the validity of the findings, interview data was analyzed thematically by categorizing student responses into categories such as learning obstacles, metacognitive experiences, and the development of creative thinking skills. Data triangulation was conducted by comparing the results of the four instruments to identify patterns of consistency or inconsistency. For instance, an increase in creative thinking scores from the test would be reinforced if the observation data demonstrates the effective implementation of the metacognitive approach, and if the questionnaire and interview reflect students' positive perceptions of learning. The final conclusion was drawn based on a synthesis of mutually supportive findings, providing recommendations for enhancing the didactic design and confirming the effectiveness of the metacognitive approach in improving students' creative thinking skills in geometry material.

Findings and Discussion

This research commenced with a study aimed at gathering data on learning impediments faced by students due to the flat building material used. The data was obtained through a mathematical creative reasoning diagnostic test administered to students. This diagnostic test is also employed in the development of didactic designs for the trial class. The findings of the identification of learning obstacles in students are presented in Figure 2, Figure 3, and are summarized in Table 6 and Table 7, which highlight the errors encountered by students in the geometry curriculum.

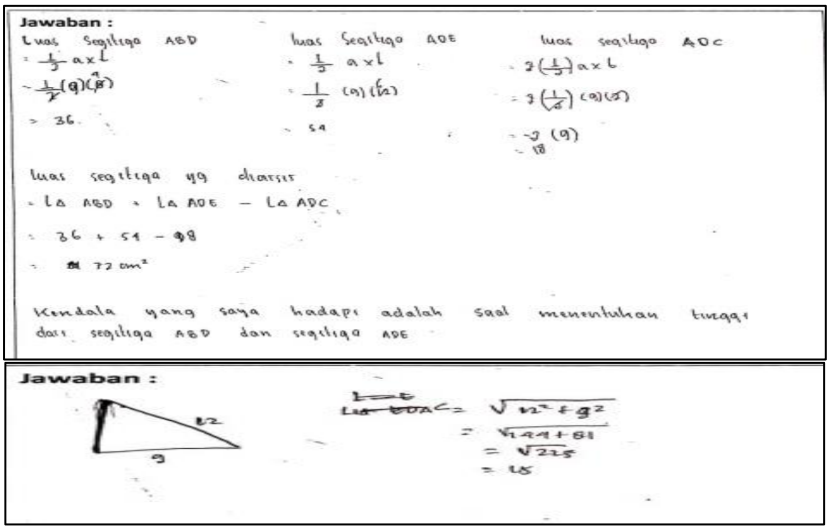


Figure 2. Student answer process

Figure 2 illustrates a series of student errors in comprehending the concept of flat shapes. These errors occur in type 1 geometry problems that include shaded areas of flat shapes. Students are incorrect in identifying the combinations of flat shapes presented in the questions. Table 6 provides a detailed analysis of the errors encountered by students while solving type 1 problems.

Table 6. Error Types, error descriptions, problem-solving strategies, and problem-solving approaches for problem type 1

Error Types	Error Description	Ways of Thinking	Ways Understanding Problems
Rules	Students lack a comprehensive understanding of flat wakes, which encompass shaded flat surfaces.	Students equate ΔABD and ΔBDF builds. Then students also equate building ΔADE and ΔAEG.	ΔABD and ΔBDF are different, and ΔADE and ΔAEG are different triangles.
	Students lack the comprehension of the interpretation of a flat, shaded building.	Students identify the height of ΔABC = 8 cm while 8 cm is the height of ΔBDF. Likewise, the height of	8 cm is the height of ΔBDF, as well as 12 cm is the height of ΔAGE.

Error Types	Error Description	Ways of Thinking	Ways Understanding Problems
		ΔCDE is identified as 12 cm while it is the height of ΔAGE.	
Errors in the use and understanding of notation/ symbols	Students lack the comprehension of the significance or absence of symbols in problem-solving.	Students do not understand the notation of the use of Δ and <.	There are different meanings of the notation Δ and <, where Δ to represent a flat triangle and < to represent angles.
Interpreting solutions	Students interpret the solution of a flat-shaded building by subtracting the entire wake from an unshaded building. This ensures that the resulting value does not align with the intended outcome.	The shaded plane shape value is explained by: Overall build area – area ΔACD.	The area of the shaded building area should be described as: Area $\Delta ABC + \Delta CDE$.

Table 6 presents various obstacles encountered by students in solving type 1 problems that pertain to the concept of Euclidean plane. The primary impediment lies in the comprehension of the equivalence rule, where students equate the type and size of flat planes without distinguishing their geometric attributes. This error suggests a fundamental lack of understanding of the concept of flat planes, particularly in the shaded region. Furthermore, students frequently misidentify the height of the triangle, erroneously considering a height of 8 cm as the height of the entire plane, when in fact it only applies to specific parts. This reflects deficiencies in the analysis of geometric elements. Regarding the utilization of notation and symbols, students fail to recognize the distinction in meaning between Δ (triangle) and < (angle), leading to errors in interpreting the problem. Additionally, errors in interpreting solutions often occur, where students miscalculate the area of the plane by employing an incorrect subtraction method. For instance, they may calculate the area of the shaded plane by subtracting the total area of the plane from the area of the unshaded plane, when in fact this approach is erroneous. This obstacle underscores a lack of comprehensive evaluation of the work process and the resulting outcomes.

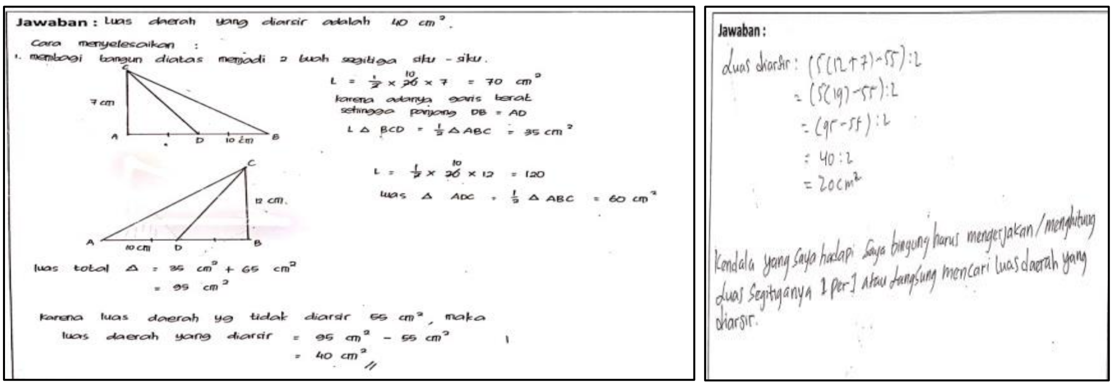


Figure 3. Student answer process

Figure 3 presents a summary of students' errors in identifying the base of the triangle. Notably, students made incorrect identifications of the compound flat shapes presented in the problem. Furthermore, it was observed that students failed to assign names to the triangles included in the questions. Additionally, students exhibited incorrect addition and multiplication operations, resulting in incorrect final results. Table 7 below provides a detailed analysis of the errors encountered by students while solving type 2 problems.

Table 7. Error Types, error descriptions, problem-solving strategies, and problem-solving approaches for problem type 2

Error Types	Error Description	Ways of Thinking	Ways Understanding Problems
Triangle Inequality Theorem	Students frequently misidentify the triangular bases of objects.	Students measure the base of triangle 1 in centimeters, dividing it into two equal segments of 20 centimeters each.	The base of triangle 1 has a length of 10 centimeters.
	Students misidentify the combined flat builds listed in the problem.	The student presumes that the shaded triangle represents only a portion of triangle 1.	The shaded region encompasses the common area shared by both unshaded triangles.
Errors in the use and understanding of notation/ symbols	Students do not assign names to the triangles included within the problem.	Students erroneously believe that naming triangles is not essential for identifying them.	Naming a triangle serves to describe its geometric shape.
Interpreting solutions	Students make arithmetic errors when adding and multiplying, resulting in incorrect final answers.	Students do not re-examine the results of the calculations performed.	Students revisit the solution to the problem.

Table 7 elucidates the challenges encountered by students in solving type 2 problems that incorporate plane area and triangle concepts. A notable obstacle emerged in comprehending the triangle rule, where students erroneously identified the length of the base of the initial triangle with its actual length. This error underscores students' limited ability to meticulously examine the available data. Furthermore, students frequently misinterpreted the shaded portion of the triangle as a mere fraction of a given triangle, failing to recognize that it represents the amalgamation of two unshaded triangles. This obstacle highlights limitations in spatial visualization. Additionally, the absence of naming or labeling plane elements rendered it challenging to discern pertinent components. Lastly, incorrect interpretations of solutions, such as addition or multiplication errors, indicate a deficiency in evaluating the calculation process. This also suggests that students tend to overlook their results, thereby failing to identify fundamental errors.

Table 6 and Table 7 separately demonstrate that students primarily encounter challenges due to a lack of conceptual comprehension, misinterpretation of geometric elements, and insufficient independent evaluation of solutions. The learning obstacles identified in Table 6 and Table 7 serve as the foundation for developing a metacognitive-based didactic design that can assist students in enhancing their understanding and sharpening their systematic thinking abilities. This approach provides a framework that enables students not only to rectify specific errors but also to gain awareness of their thought processes. Consequently, students are guided to adopt a more structured approach to problem-solving, closely monitoring their work steps, and evaluating solutions. This design empowers students to overcome previously identified weaknesses. The metacognitive-based didactic design resulting from the identified obstacles is presented in Table 8.

Table 8. Didactic design with a metacognitive approach

Stages	Metacognitive approach strategies
Orientation to problems	Students are advised to read the questions thoroughly and meticulously record pertinent data.
	Employing lead questions such as: “What is your understanding of this issue?”
Planning to overcome problems	Facilitate students in formulating goals and problem-solving strategies.
	Students create a list of solution plans with the help of mind maps.
Mastery of prior knowledge (concept of creativity)	The instructor offers exercises that assist students in identifying geometric symbols, specifically distinguishing the triangle (Δ) from the angle symbol (<).
	Students are asked to label the elements of the field to aid further analysis.
Realization of plans	Students receive comprehensive instruction on the sequential steps involved in problem-solving, encompassing the identification of geometric elements such as height, base, and area.
	Students are instructed to verify the data, ensuring that the interpretation of height or area aligns with the intended geometric element.
Evaluation of results	Students are required to compare the final outcome with the preceding steps to identify any errors.
	The lecturer provides feedback on student solutions and discusses common mistakes as collaborative learning.

The results of the metapedadidactic analysis focused on the application of didactic design based on a metacognitive approach in classroom learning to evaluate its effectiveness in improving students' creative thinking skills. At this stage, the previously prepared didactic design was implemented through three main stages: planning, monitoring, and evaluation, with each stage designed to overcome the learning obstacles previously identified in Table 6 and Table 7.

In the implementation phase of the didactic design based on the metacognitive approach in the classroom, the learning process becomes more structured. During the learning process, students are engaged in reading the questions thoroughly and are encouraged to meticulously record relevant data. To facilitate initial comprehension, the instructor facilitates a discussion about the contextual background of the questions, introducing fundamental geometric elements such as the height of a triangle, its base, and geometric symbols (e.g., Δ for a triangle and < for an angle). Students are also tasked with labeling each component of the geometric elements, for instance, naming a specific triangle as “triangle A” or “triangle B,” thereby enhancing the identification process. Simultaneously, monitoring is conducted as students complete the questions, guided by a systematic step-by-step worksheet. This process entails identifying geometric elements, distinguishing shaded and unshaded regions of the plane, and applying relevant formulas. Students are also instructed to verify the accuracy of their work steps, ensuring that the data and methodology employed are appropriate. In this session, the instructor encourages small group discussions to foster flexibility of thought and exploration of alternative solutions. Subsequently, after students have completed the questions, they are encouraged to compare their solutions with those of their group members and engage in a discussion about the variations in results. The instructor provides immediate feedback on student work, identifying common errors and offering recommendations for improvement. Furthermore, students are provided with the opportunity to reflect on their thought process, verifying that the steps taken align with the requirements of the problem. This iterative process fosters a more interactive, reflective, and immersive learning environment, which significantly enhances students' creative thinking abilities.

The implementation of metacognitive-based didactic design fostered an interactive and reflective classroom environment. Students exhibited notable transformations in their creative thinking abilities. Initially, they exhibited uncertainty regarding the appropriate steps, but over time, they demonstrated a growing sense of confidence in formulating strategies and exploring diverse approaches to problem-solving. Group discussions became more dynamic, as students engaged in the exchange of creative solutions and embraced novel ideas. Additionally, lecturers assumed a facilitative role, encouraging students to become cognizant of their own thought processes.

The implementation of this design yielded positive outcomes. Students who previously frequently made errors in comprehending geometric elements or utilizing symbols demonstrated greater accuracy in identifying crucial data and interpreting solutions. Additionally, they exhibited improvements in the fluency of their thinking by systematically generating novel ideas, demonstrating flexibility in attempting various approaches, and presenting more creative solutions. The classroom environment that fosters reflection and self-evaluation enhances students' awareness of their thought processes, which is the primary determinant of enhancing their creative thinking abilities.

The metapedadidactic analysis stage substantiates that the metacognitive-based didactic design employed in education can create an environment conducive to the development of creative thinking skills. This approach not only addresses students' learning challenges but also cultivates a more critical, reflective, and innovative mindset in solving geometric problems.

The third phase in implementing a didactic design based on a metacognitive approach involves a retrospective analysis, which aims to assess the overall learning outcomes after the metapedadidactic stage has been executed. At this stage, data from assessments, questionnaires, observation sheets, and interviews are meticulously analyzed to evaluate the efficacy of the implemented didactic design. Table 9 presents the results of students' creative thinking skills test before and after the implementation of a didactic design based on a metacognitive approach, based on the analysis of the results of Test 1 (before) and Test 2 (after).

Table 9. Results of students' critical thinking skills assessment

Indicators	Before (Test 1)	After (Test 2)	Evolution of Critical Thinking Abilities
Fluency	3.2	4.1	0.9
Flexibility	3.0	4.0	1.0
Novelty	2.8	3.8	1.0

The analysis of students' creative thinking skills before and after the implementation of a didactic design based on the metacognitive approach (Table 9) revealed a significant enhancement in all primary indicators, including fluency, flexibility, and novelty. Prior to the implementation of the didactic design, the average score for students' fluency was 3.2, indicating that they were still limited in generating ideas spontaneously and systematically. Subsequently, this score increased to 4.1, reflecting a superior ability to organize and convey ideas swiftly and effectively. The flexibility indicator, which had an average score of 3.0 before the implementation, also rose to 4.0. This suggests that students are now better equipped to employ diverse problem-solving strategies and adapt to various approaches. Furthermore, the novelty indicator increased from 2.8 to 3.8, indicating that students are beginning to produce more creative and distinctive solutions in problem-solving, although there is still room for further improvement.

The improvement in all these indicators suggests that metacognitive-based didactic design not only effectively assists students in comprehending geometry materials but also fosters creative and deep thinking. This approach has been demonstrated to enhance students' metacognitive abilities, including planning, monitoring, and evaluation, which directly contribute to their capacity for flexible and innovative thinking. This finding reinforces the notion that integrating didactic design with a metacognitive approach is a potent learning strategy for enhancing students' overall creative thinking capabilities.

The subsequent analysis is focused on the questionnaire data. The results of the questionnaire designed to assess students' perceptions of learning through a metacognitive approach were provided to 28 students. The average score for each category is presented in Table 10 as follows.

Table 10. Results of the student perception questionnaire

Aspect	Average score
Understanding the material	4.2
Creative thinking skills	4.1
Challenges encountered in the learning process	3.8
Efficacy of the metacognition approach	4.3

Based on Table 10, the results of the student perception questionnaire on learning with a metacognitive approach reveal a generally positive perception among students. The average score in each category falls within the desirable range (above 4 on a scale of 1-5). The table above (rearrange the order) provides insights into the effectiveness of the metacognitive approach: (1) Understanding the Material: Students (Average Score: 4.2) perceive that learning with a metacognitive approach enhances their comprehension of geometry material. This suggests that the didactic design effectively improves conceptual understanding. (2) Creative Thinking Skills: Students (Average Score: 4.1) demonstrate a favorable perception of the approach's role in developing their creative thinking abilities. This indicates that the metacognitive approach encourages students to adopt flexible thinking and generate innovative solutions. (3) Difficulties Experienced: While the score in this category (Average Score: 3.8) is slightly lower compared to the others, it suggests that some students encounter challenges during learning. However, a score close to 4 indicates that the metacognitive approach has assisted most students in overcoming these difficulties. (4) Effectiveness of the Metacognitive Approach: This category (Average Score: 4.3) received the highest score, indicating that students were highly aware of the benefits of the metacognitive approach in learning. They recognized that this approach enhanced their awareness of the thinking, planning, and evaluation processes involved in problem-solving.

Overall, the findings of the questionnaire support the efficacy of the metacognitive approach and didactic design in enhancing students' learning experiences. Particularly, these approaches have demonstrated success in fostering students' understanding of the material and cultivating creative thinking abilities. While some challenges persist, these data serve as valuable insights for refining the learning design to further enhance students' ability to overcome learning difficulties. Furthermore, the outcomes of observations of the application of learning (didactic design) employing a metacognitive methodology are presented in Figure 4 and Figure 5.

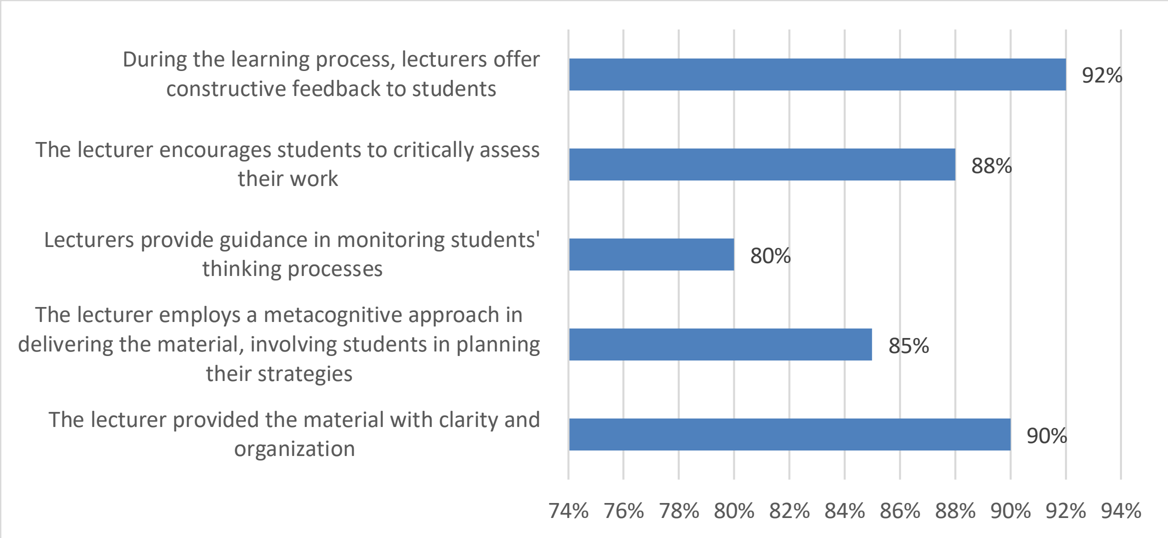


Figure 4. Percentage of achievement of lecturer's activities

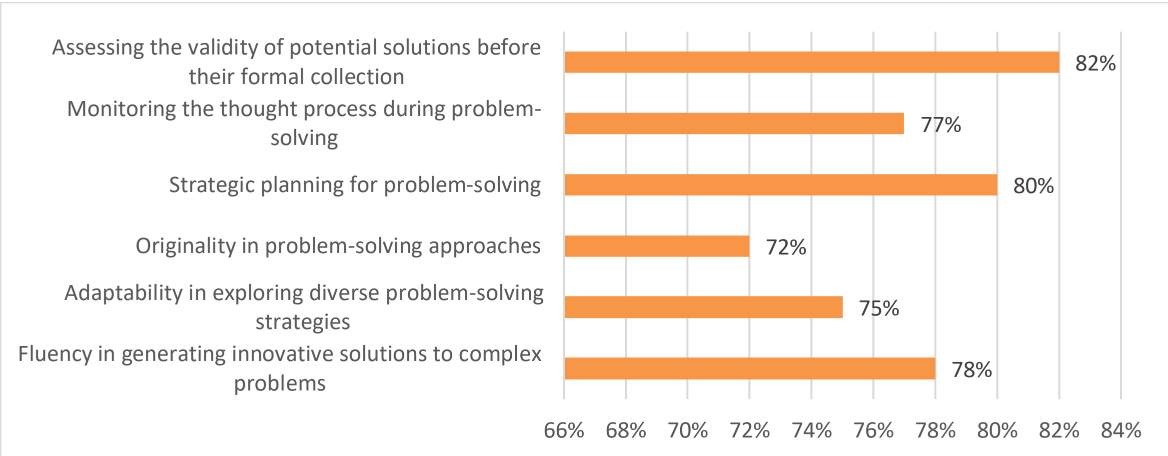


Figure 5. Percentage of achievement of student's activities

Based on Figure 4 and Figure 5, which provide observations of the achievement of lecturer and student activities, it can be concluded that the implementation of didactic design with a metacognitive approach is effective, resulting in a high level of achievement in most indicators.

From the lecturer's perspective, the indicator with the highest achievement is the provision of constructive feedback (92%). This demonstrates that the lecturer actively provides direction and evaluation that supports the student's learning process. Additionally, the delivery of clear and systematic material is also commendable (90%), ensuring that students grasp the geometry concept in a structured manner. Other indicators, such as the consistent integration of a metacognitive approach in learning (85%) and encouragement to evaluate answers (88%), also indicate the lecturer's success. However, guidance in monitoring students' thinking processes (80%) presents an area for improvement.

From the student's perspective, the indicator with the highest achievement is the evaluation of answer results before their collection (82%). This demonstrates that most students are able to reflect on and review their answers. Furthermore, planning steps to solve problems (80%) suggests that students are beginning to develop strategies for problem-solving. However, several indicators related to creativity, such as fluency in generating ideas (78%), monitoring thought processes (77%), flexibility in trying different strategies (75%), and novelty of solutions (72%), exhibit lower levels of achievement. This suggests that some students still require assistance in applying creative thinking processes independently.

Overall, the data indicates that lecturers have effectively supported metacognitive learning, and students generally responded favorably. However, further guidance is necessary, particularly in areas such as flexibility and novelty in creative thinking. To enhance the test data, questionnaires, observation sheets, and interviews were conducted to delve into students' responses to the application of didactic design with a metacognitive approach. An excerpt from an interview conducted by the researcher with one of the respondents, MH1, is provided below:

Researcher: What do you think about the learning approach applied? Does it help you understand the material and improve your creative thinking skills?

MH1: In my opinion, this learning is very helpful. Usually, I only focus on solving problems without thinking about the most effective strategy. However, with this method, I am taught to plan, monitor, and evaluate the answers. This makes me more aware of my own thinking process. Although it was difficult at first, over time I feel more confident, especially when asked to try new ways to solve problems.

Researcher: Do you think this approach helps you solve difficult geometry problems? If so, how?

MH1: Yes, I find this approach very helpful, especially when asked to make a plan before starting to work on the problem. In the past, I often tried to solve problems directly without a strategy, but now I am taught to think about the steps first. In addition, I learn to monitor the answers during the process, so I know if there are mistakes. Finally, evaluating the answers makes me sure that the results I give are correct or at least better than before.

Researcher: Do you feel more creative in solving problems after this learning?

MH1: I feel more creative because I am often asked to try various ways to solve a problem. It was difficult at first, but with the help of lecturers and practice, I began to think more flexibly and sometimes found ways that I had never thought of before.

The research findings revealed that students encountered three primary challenges during their learning experience employing a metacognitive approach: learning obstacles, metacognitive experiences, and the development of creative thinking abilities. Initially, students encountered difficulties in formulating strategies and monitoring the problem-solving process. This was primarily attributed to their accustomed habit of answering questions directly without considering the necessary steps. However, the instructor's support through guidance and feedback facilitated the students' overcoming of these obstacles. Over time, students became accustomed to utilizing metacognitive strategies, such as planning steps to solve problems, monitoring their responses, and evaluating the final outcomes. This demonstrated an enhanced awareness of their own thought processes. Students also reported an increase in their confidence in exploring various problem-solving approaches, indicating a development in thinking flexibility. Some students even mentioned that this approach encouraged them to generate more distinctive or unique solutions compared to their usual practices, although the novelty aspect required further reinforcement through more intensive practice.

The interviews provided in-depth insights into students' experiences during learning, which significantly strengthened the results of tests, questionnaires, and observations. The initial challenges encountered by students underscored the necessity for intensive mentoring during the early stages of implementing the metacognitive approach. However, these findings also demonstrated substantial advancements in students' ability to plan, monitor, and evaluate their thinking processes, aligning with the principles of metacognition. Furthermore, the development of creative thinking skills, such as flexibility and novelty, was evident in students' capacity to explore diverse problem-solving approaches and generate novel solutions. These findings suggest that learning with metacognitive-based didactic design not only enhances learning outcomes but also fosters students' mindsets that are more structured and creative. The integration of all research instruments provides a comprehensive understanding of the approach's effectiveness in supporting students' holistic development, encompassing both cognitive and creative thinking abilities. These findings also offer recommendations for enhancing the didactic design by increasing exercises that promote students' novelty in problem-solving approaches.

The investigation shows that both conceptual and procedural difficulties are present in students' difficulties with geometry problems involving triangles. The inability to differentiate between different triangle kinds according to their characteristics is one of the main problems. Key concepts like the sum of angles in a triangle and the proper labelling of geometric features like the base or special lines (like height or medians) within combined shapes are frequently not understood by students.

Several factors have been identified as contributing to these challenges: conceptual difficulties arise from a lack of understanding of prerequisite material, such as angles, and weak connections between mathematical concepts, such as proportional reasoning. Additionally, students often make procedural errors, including incorrect addition or multiplication, due to poor procedural fluency. This fluency can be further exacerbated by misinterpreting the problem statement or hastily completing tasks without careful validation of results.

A study conducted by Musfiratul et al. (2023) underscores the prevalence of students' inability to grasp fundamental geometric concepts. Consequently, they resort to superficial strategies, such as guessing answers or recognizing figures solely based on their visual appearance, without engaging in deeper analytical reasoning. This finding aligns with van Hiele's theory, which posits that a significant portion of students remain at the visualization stage (level 1), primarily identifying shapes based on their visual attributes rather than comprehending their intrinsic properties.

The following is a summary of students' errors in identifying the base of the triangle. Additionally, students were incorrect in identifying the compound flat shapes listed in the problem. Furthermore, it was discovered that students failed to assign names to the triangles included in the questions. Furthermore, it was found that students made incorrect additions and multiplications, resulting in an incorrect final answer.

In addressing the question of dividing the rectangle into two equal parts, students demonstrate a lack of creativity and fail to break free from the conventional flat building pattern. Students are inextricably linked to the shape and image of the flat building, which mirrors the existing questions. The student misplaced the circled box on the picture, resulting in the non-formation of a cube. These obstacles can manifest as misconceptions, difficulties in visualization, or a lack of comprehension of geometric properties (Elbehary et al., 2023; Banson et al., 2023; Kusno & Sutarto, 2022). Naturally, this is influenced by prior knowledge acquired through a metacognitive approach, which refers to the knowledge possessed by individuals or groups prior to engaging in a specific learning or task (Stanton et al., 2021; Kostons & van der Werf, 2015).

Didactical Design Research (DDR) holds significant importance in the realm of education for several fundamental reasons. Firstly, it serves as a valuable tool for educators to identify and address the challenges encountered by students during the learning process. By employing this research-driven approach, educators can design more effective and contextually relevant interventions that directly enhance the quality of education.

However, there are several issues related to prior knowledge in the context of education and learning, namely that each student brings a different level of prior knowledge into the classroom. This can be challenging for lecturers, as they need to manage different levels of knowledge within one class. Furthermore, uneven prior knowledge among students can create gaps in classroom understanding. Students who have a better understanding of a particular topic may feel bored or overly challenged, and thirdly, lecturers need to have effective strategies to link previous knowledge to students with the new material taught. This will make it easier for students to understand new concepts.

Furthermore, research demonstrates that promoting self-regulated learning and effectively managing cognitive load enhance students' ability to apply their prior knowledge more effectively. To engage students with diverse knowledge levels in meaningful ways, educators must develop instructional strategies that simultaneously reduce cognitive overload and foster active engagement, such as group discussions and problem-solving exercises. These findings underscore the importance of meticulously planning lessons that incorporate past experiences into the learning process, ensuring that all students can achieve success regardless of their current learning level (Aslanov & Guerra, 2023; Dong et al., 2020).

The learning trajectory phase of creative thinking geometry development in education encompasses the acquisition of the ability to formulate problems, discern unusual connections between mathematical concepts, and generate innovative solutions. This study identifies five hierarchical phases within this phase: (1) Orientation to the Problem: Students begin by comprehending the problem at hand. (2) Plan to Overcome the Problem: Students develop a plan to address the problem effectively. (3) Realization of the Plan: Students implement their plan and execute the necessary actions. (4) Mastery of Previous Knowledge/Concepts of Mathematical Creativity: Students reinforce their understanding of previous knowledge and concepts related to mathematical creativity. (5) Evaluation of Results Obtained: Students assess the outcomes of their efforts and evaluate the effectiveness of their creative solutions. Throughout this learning trajectory, students engage in metacognition, reflecting on their learning process. This includes evaluating their planning, executing actions, and selecting creative ideas.

Based on this explanation, it is crucial for students to possess sufficient prior knowledge to facilitate the acquisition of new knowledge. Additionally, it is essential to design the learning process in a manner that is engaging and meaningful, thereby enhancing students' long-term memory retention. For researchers, the subsequent step involves leveraging their existing knowledge to support the learning of geometry. Students aspire to design effective and meaningful learning experiences. Covey (2013) referred to this ability as "inside-out," emphasizing that the internal state of an individual can influence the external environment, particularly in the context of learning. This encompasses the learning environment, student characteristics, and learning objectives. It is imperative to accommodate individual differences in learning styles, abilities, and interests, as well as the provision of appropriate feedback to students within the didactic design. This approach is fundamental to creating an effective and relevant learning experience for students.

Overall, this study demonstrates that a didactic design based on a metacognitive approach not only addresses learning obstacles but also substantially enhances students' creative thinking abilities. By aligning with previous research findings, the outcomes of this study make significant contributions to educational literature, particularly in the realm of learning strategies aimed at fortifying creative thinking skills within higher education institutions.

Conclusion

The developmental learning trajectory phase in learning describes the progression of students' understanding and mastery of the concept of creative thinking in geometry. This includes the ability to formulate problems, identify unusual relationships between mathematical concepts, and generate innovative solutions.

Based on the research findings, it can be concluded that the current learning outcomes are concerning. Specifically, students exhibit limited creativity and are often trapped in a conventional approach to problem-solving. Their initial knowledge lacks the analytical skills to dissect flat shapes into their component parts and subsequently synthesize those parts into complex forms. This lack of analytical and synthetic abilities hinders their ability to comprehend the relationships between various flat shapes.

Furthermore, the learning trajectory of creative mathematical thinking is structured into five hierarchical phases: orientation to problems, planning to overcome problems, realization of plans, mastery of prior knowledge (concept of creativity), and evaluation of results.

The didactic design developed can effectively address the obstacles faced by students. It is crucial for students to possess sufficient initial knowledge to facilitate the acquisition of new knowledge. Additionally, incorporating elements that make the learning process engaging and meaningful is essential to enhance students' long-term memory retention.

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